Elasticity and Curvature of Discrete Choice Demand Models

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Motivation

- *Demand curvature* is important for understanding price, quantity, and welfare effects of cost and policy changes, including taxation/exchange rate pass-through, efficiency gains of mergers, or nominal *vs.* real adjustment costs.
- Current empirical models are capable of "reasonable" substitution patterns.
- We don't know how modeling assumptions may limit estimates of demand curvature in discrete choice models.
- Is it possible to estimate robust demand curvatures using common, familiar methods?
- *(How did we get here? – Perhaps at the end if we have time...)*

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These substitution patterns drive answers to many questions of interest — e.g., the sizes of markups or outcomes under a counterfactual merger. However, other kinds of counterfactuals can require flexibility in other dimensions. For example, "pass-through" (e.g., of a tariff, tax, or technologically driven reduction in marginal cost) depends critically on second derivatives of demand. It is not clear that a mixed-logit model is very flexible in this dimension.

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Model Specification and Curvature Restrictions

- Non-parametrically methods deal with "reasonable" shape restrictions at a high computational cost, limiting their applicability.

  – Compiani (2022); Magnolfi, McClure & Sorensen (2022)

- We focus on mixed-logit (ML) demand because the framework:
  - Can accommodate many products.
  - Is a workhorse model for research and policy.
  - Can approximate any random utility model.

    – McFadden & Train (2000)

- We offer a framework for researchers to avoid restricting the range of estimable demand curvature, and thus the predictions of our model on pass-through.

Contributions

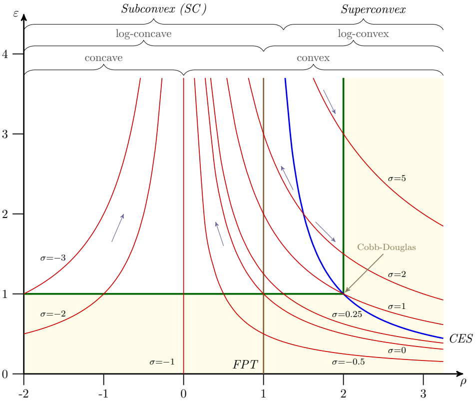
1. Document the sources of demand curvature in **unit demand DCM** consistent with utility maximization.

2. **The Demand Manifold**: Connecting elasticity and curvature and show how common modeling assumptions restrict their relationship.

3. Modify the ML model to generate flexible estimates of both demand elasticity and curvature.
   - Quasilinear utility (shape of price R.C. distribution).
   - Income effects (shape of income subfunction).

4. Empirical evidence – Flexibility is economically meaningful:
   - Identification: Monte Carlo simulation (Q-L *vs.* Box-Cox).
   - Uniform pricing: Standard approaches bias consumer welfare evaluations (IRI: RTE-Cereal).

Demand Manifold

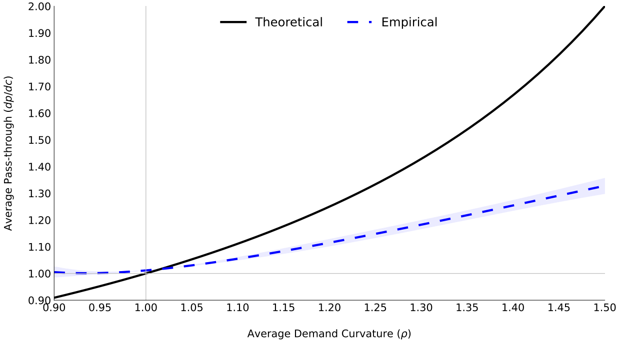


- Elasticity (ε) and curvature (ρ) are connected through the necessary and sufficient conditions of profit maximization (Mrázová-Neary, 2017).

$$p(q) + q \cdot p_q(q) = p(q)\left[1 - \frac{1}{\varepsilon(q)}\right] = c > 0 \iff \varepsilon(q) \equiv -\frac{p \cdot q_p(p)}{q(p)} > 1\,,$$

$$2p_q(q) + q \cdot p_{qq}(q) = p_q(q)\,[2 - \rho(q)] < 0 \iff \rho(q) \equiv \frac{q(p) \cdot q_{pp}(p)}{[q_p(p)]^2} < 2\,,$$

Pass-Through: Monopoly *vs.* Oligopoly



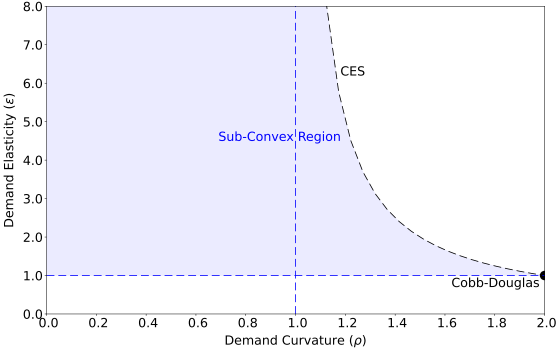
- Markups (CRS single-product monopoly/oligopoly):

$$\frac{p-c}{p} = \frac{1}{\varepsilon} \quad ; \quad \frac{p-c}{p} = \frac{\theta}{\varepsilon}$$

- Pass-through rate (Cournot, 1838 / Weyl-Fabinger, 2013):

$$\frac{dp}{dc} = \frac{1}{2-\rho(q)} \quad ; \quad \frac{dp}{dc} = \frac{1}{1+\theta(1-\rho)} .$$

Demand Subconvexity



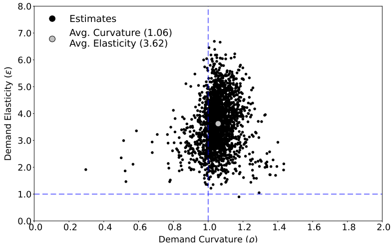
- CES is the *only case* where (ε, ρ) invariant to price:

$$\rho^{CES} = 1 + \frac{1}{\varepsilon^{CES}} \ .$$

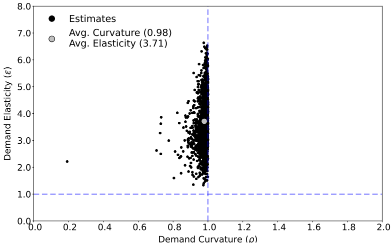
- CES is a useful limiting case which defines the area of "sub-convex" demand $\Leftrightarrow$ Marshall's *Second Law of Demand* $\Rightarrow$ Single-product oligopoly equilibrium exists (Caplin & Nalebuff, 1991).

Nevo's Elasticity and Curvature Estimates

▸ Estimates



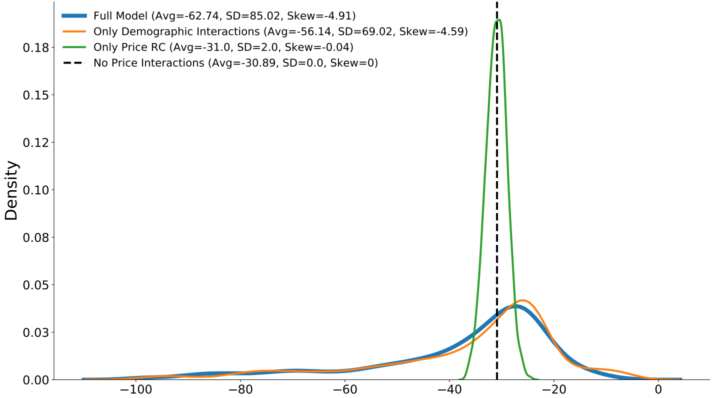
(A) Full Model



(B) Multinomial Logit

- Use Nevo's simulated breakfast cereal data to explore model predictions for different preference specifications.
- Comparing the full R.C. and ML model suggests that model specification matters for pass-through analysis.
- Price R.C. and price interactions also appear to increase the range of demand curvature estimates (not reported).
- The average elasticity and curvature estimates do not vary too much but their distributions change dramatically.

Distributions of Price Sensitivity



- Including price-demographic interactions lead to very asymmetric empirical distributions of individual demand slopes.
- MNL does not allow for any price response heterogeneity.

General Price Distribution

- We present general manifold expressions.

- We first consider the following generalization of Nevo's model where $\Phi(0,1)$ is a non-necessarily symmetric distribution:

$$u_{ij} = x_j \beta_i^\star + f_i(y_i, p_j) + \xi_j + \epsilon_{ij}\,, \qquad i \in \mathcal{I},\ j \in \mathcal{J},\ \epsilon_{ij} \sim \text{EV1}\,,$$

$$\beta_i^\star = \beta + \sigma_x \nu_i\,, \qquad \nu_i \sim N(0, I_n)\,,$$

$$f_i(y_i, p_j) = \alpha_i^\star \big(y_i - p_j\big) = \big(\alpha + \sigma_p \phi_i\big) \times \big(y_i - p_j\big)\,, \quad \phi_i \sim \Phi(0,1)\,,$$

- Choice of mixing distribution is an integral part of model specification.

  – McFadden and Train (2000)

Logit Demand

- Utility maximization: Individual i purchases her preferred product j if:

$$q_{ij}(p) = \mathbf{1}\Big(u_{ij} \geq u_{ik}\,,\ \forall k \in \{0, 1, \dots, J\}\Big),$$

- Because of i.i.d. EV1 of ϵ_{ij}, individual i's choice probability of product j is:

$$\mathbb{P}_{ij}(p) = \frac{\exp\left(x_j \beta_i^\star - \alpha_i^\star p_j + \xi_j\right)}{\sum_{k=0}^{J} \exp\left(x_k \beta_i^\star - \alpha_i^\star p_k + \xi_k\right)}\,,$$

Intermediate Results

- Consider a measure $G(i)$ of heterogeneous individuals. Total demand for product j is:

$$Q_j(p) = \int_{i \in \mathcal{I}} \mathbb{P}_{ij}(p)\, dG(i)\,.$$

- Derivatives of the utility's price subfunction (general case):

$$f'_{ij} = \frac{\partial f_i(y_i, p_j)}{\partial p_j}\,, \qquad \text{and} \qquad f''_{ij} = \frac{\partial^2 f_i(y_i, p_j)}{\partial p_j^2}\,.$$

- Consider the Bernouilli distribution (choice of one product):

$$\begin{aligned} \mu_{ij} &= \mathbb{P}_{ij}\,, \\ \sigma^2_{ij} &= \mathbb{P}_{ij}(1 - \mathbb{P}_{ij})\,, \\ sk_{ij} &= \mathbb{P}_{ij}(1 - \mathbb{P}_{ij})^2 - \mathbb{P}^2_{ij}(1 - \mathbb{P}_{ij}) = \sigma^2_{ij}(1 - 2\mathbb{P}_{ij})\,. \end{aligned}$$

Main Results

- Demand elasticity, curvature and manifold are:

$$\varepsilon_j(p) = -\frac{p_j}{Q_j(p)} \int_{i \in \mathcal{I}} f'_{ij} \cdot \sigma^2_{ij} \, dG(i) \, ,$$

$$\rho_j(p) = \int_{i \in \mathcal{I}} \mu_{ij} \, dG(i) \times \frac{\int f''_{ij} \cdot \sigma^2_{ij} \, dG(i) + \int \left(f'_{ij}\right)^2 \cdot sk_{ij} \, dG(i)}{\left[\int f'_{ij} \cdot \sigma^2_{ij} \, dG(i)\right]^2} \, .$$

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- Demand manifold is therefore:

$$\rho_j[\varepsilon_j(p)] = p_j^2 \cdot \frac{Q_j(p)}{\varepsilon_j^2(p)} \cdot \left[\int f''_{ij} \cdot \sigma^2_{ij}\, dG(i) + \int \left(f'_{ij}\right)^2 \cdot sk_{ij}\, dG(i)\right].$$

MNL: $\alpha_i^\star = \alpha\,;\beta_i^\star = \beta$

- Demand elasticity and curvature for MNL are:

$$\varepsilon_j(p) = \alpha p_j \left(1 - \mathbb{P}_j\right),$$

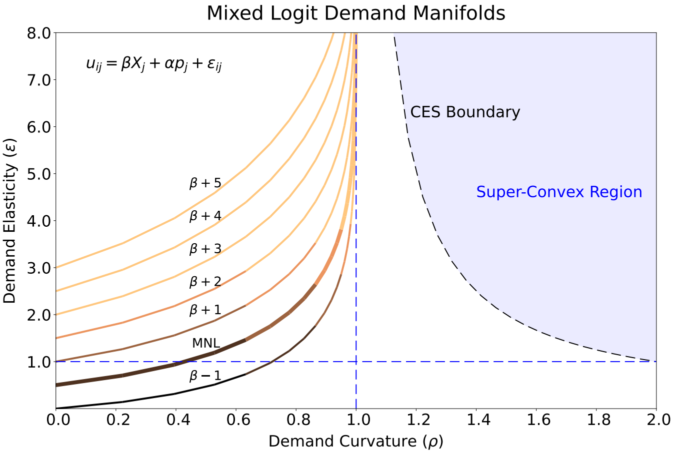
$$\rho_j(p) = \frac{1 - 2\mathbb{P}_j}{1 - \mathbb{P}_j} < 1\,.$$

- And the demand manifold is:

$$\rho_j(p) = \frac{\alpha p_j \left(1 - 2\mathbb{P}_j\right)}{\varepsilon_j(p)}\,.$$

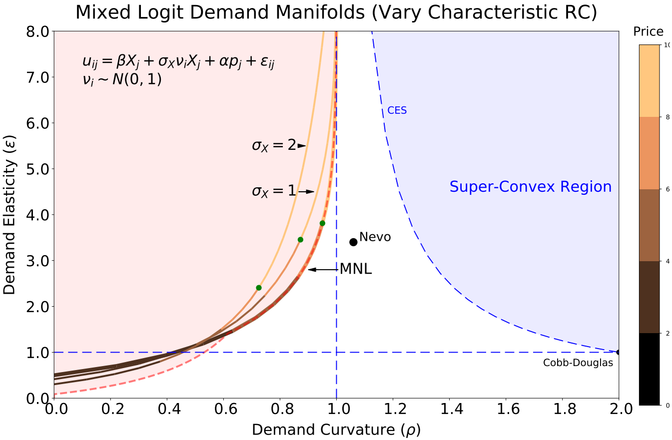
- MNL is *always* log-concave (it imposes incomplete pass-through).

MNL: α ; β



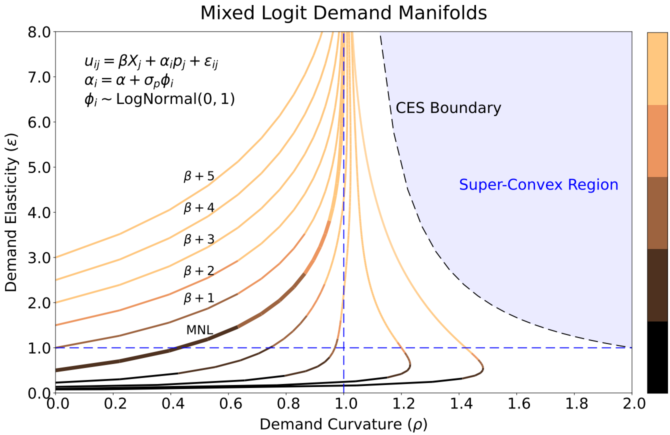
- Consider a monopolist: one inside good with utility $u_{ij} = 1 - 0.5p_j + \epsilon_{ij}$.
- Logit manifold. For any given curvature, a larger market share of the product makes demand less elastic or, alternatively, for any given elasticity, a larger market share reduces demand curvature.

MNL with Attribute Heterogeneity: $\alpha\,;\beta_i^\star$



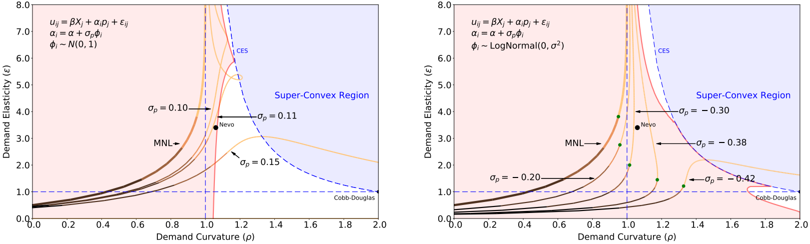
- Random coefficients of attributes allow for flexible substitution patters.
- Random coefficients of attributes cannot generate more than complete pass-through if log-concave distributed. – Caplin-Nalebuff (1991)

MNL with Price Heterogeneity: $\alpha_i^\star$; β_i



- Price random coefficients to expand the range of estimable curvatures within a unit demand discrete choice model.

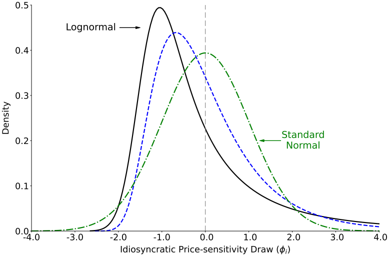
The Shape of the Mixing Distribution

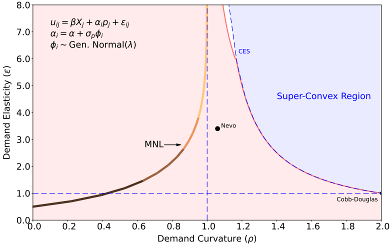


(A) Normal (B) Lognormal

- Skewness of the mixing distribution allows for greater curvature.
- If demand estimates fall in the sub-convex region, a common cost increase for a multiproduct firm results in a markup reduction for *all* its products and not only for a subset (when some of these estimates fall into the super-convex region).
- Symmetric R.C. distributions may restrict the range of estimable curvatures and bias demand elasticity estimates upwards.

Discussion





- Choosing the shape of the distribution is not trivial — Combining normal and lognormal allows us to cover nearly all the set of sub-convex demands.
- Asymmetric distributions have been used to ensure that all individuals show responses to price or attributes of the same sign. – Train (2009)
- We can capture nearly all the sub-convex region with a flexible distribution of price sensitivity:
- CES now rationalized by the distribution of price-sensitivities (α_i).

BLP: Discrete Choice with Income Effects

- Remember that Nevo's quasi-linear preferences implied the following utility subfunction:

$$f_i(y_i, p_j) = \alpha\big(y_i - p_j\big).$$

- *BLP* does not include random coefficients on prices but rather they allow for expenses in other products to depend on income:

$$f_i(y_i, p_j) = \alpha \ln\big(y_i - p_j\big).$$

- An obvious generalization involves the use of the Box-Cox Transformation:

$$f_i(y_i, p_j) = \alpha\big(y_i - p_j\big)^{(\lambda)} = \begin{cases} \alpha \dfrac{\big(y_i - p_j\big)^{\lambda} - 1}{\lambda}, & \text{if } \lambda \neq 0, \\ \alpha \ln\big(y_i - p_j\big), & \text{if } \lambda = 0. \end{cases}$$

BLP99 Approximation

- For practical reasons most applications including income effects follow the specification of $BLP99$, which is close to a Maclaurin first order approximation:

$$f_i(y_i, p_j) \simeq \alpha \ln y_i - \frac{\alpha}{y_i} p_j \approx -\alpha_i^\star p_j \, .$$

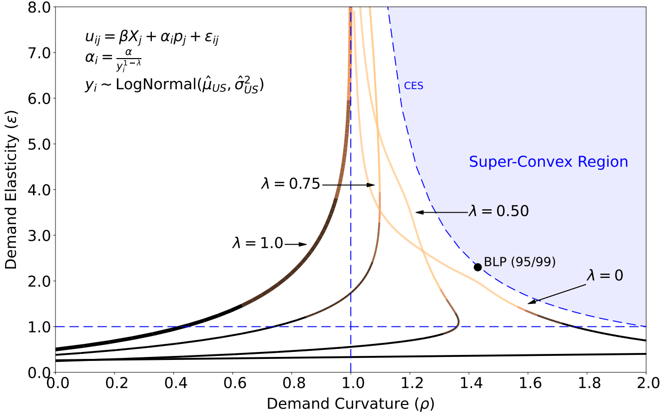
- More generally, using Box-Cox:

$$f_i(y_i, p_j) = \alpha \big(y_i - p_j\big)^{(\lambda)} \simeq \alpha y_i^{(\lambda)} - \frac{\alpha p_j}{y^{1-\lambda}} \, .$$

- Thus, we allow data to pin down the strength of income effects through more flexible price sensitivity formulation that is still (roughly) consistent with **utility maximization** (Roy's Identity).

- $\lambda = 0 \rightarrow BLP99$; $\lambda = 1 \rightarrow$ quasi-linear (MNL).

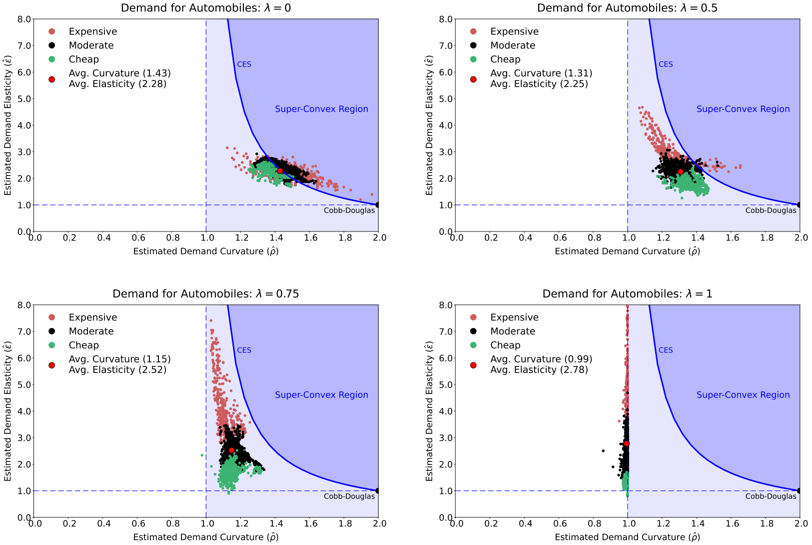
Income Effects: Box-Cox Transformation, λ



- Income effects play the same role than the distribution of price random coefficient in expanding the range of estimable demand curvatures.
- For any given λ the resulting pass-through estimate is critically determined by the empirical income distribution.

BLP99: Effects by Price Segments

By Origin



Demand for Automobiles

Table: Income Effects, Markups, and Pass-Through Rates

	$\lambda = 0$		$\lambda = 0.5$		$\lambda = 0.75$		$\lambda = 1$	
ε	2.28	(0.26)	2.25	(0.48)	2.52	(1.01)	2.73	(2.05)
ρ	1.43	(0.08)	1.31	(0.07)	1.15	(0.05)	0.99	(0.01)
Markup (%)	44.41	(5.26)	46.25	(8.77)	44.48	(13.77)	48.12	(20.55)
Pass-Through (%)	178.99	(18.33)	145.91	(16.38)	117.90	(7.27)	99.41	(0.01)

- Quasilinear MNL specification ($\lambda = 1$; $\alpha_i^\star = \alpha$) always predicts full pass-through at the cost of excessively elastic demand.
- BLP99 specification ($\lambda = 0$; $\alpha_i^\star = \frac{\alpha}{y_i}$) leads to larger pass-through rates.
- Averages differ but dispersion for elasticity and markups are also more pronounced for quasilinear preferences while the opposite is true for curvature and pass-through for BLP99 – important heterogeneous implications of counterfactual analysis.

Summary of Theoretical Results

- Robust estimates of demand curvature requires flexible specification of price interactions with consumer heterogeneity.
- A way of doing so is allowing price sensitivity to vary with observed demographics, e.g., income.
- Flexible interaction of demographics with prices is useful to account for pass-through in oligopoly with a parsimonious one-parameter transformation (Box-Cox) that modulates curvature:

$$u_{ij} = x_j\beta_i^\star + f_i(y_i, p_j) + \xi_j + \epsilon_{ij}$$

$$\textbf{Income Effects:}\quad f_i(y_i, p_j) = \alpha(y_i - p_j)^{(\lambda)}$$

$$\textbf{Quasilinear:}\quad f_i(y_i, p_j) = \alpha D_i^{(\lambda)} p_j$$

- How to identify λ?

Monte Carlo: Data Generating Process

$$u_{ijt} = \underbrace{\beta_0}_{\substack{\text{Common}\\\text{Across}\\\text{Consumers}}} + \underbrace{\sum_{k=1}^{K} (\beta_X^k + \sigma_X^k \nu_i^k) x_{jt}^k}_{\substack{\text{Idiosyncratic}\\\text{Characteristic Tastes}}} - \underbrace{\alpha \cdot p_{jt} \cdot y_{it}^{\lambda-1}}_{\substack{\text{Idiosyncratic}\\\text{Price Sensitivities}}} + \xi_{jt} + \epsilon_{ijt} \,,$$

1. Indirect utility with income effects: $J=20$, $T=100$, $I=1000$
2. Two $(K=2)$ observable attributes (x^k) with common (β_X^k) and idiosyncratic (σ_X^k) valuations.
3. Income y_{it} iid LogNormal as in Andrews, Gentzkow & Shapiro (2017) + time variation.
4. Researcher knows cost shocks ω_{jt} and marginal cost function (log-linear).
5. Solve for equilibrium prices s.t. inside share $= 20\%$ and $\overline{\varepsilon}=2.5$ $\Rightarrow (\beta_0, \alpha)$.

Identification

- Identify σ_X^k via Gandhi & Houde (2020) *Differentiation IVs*:

$$Z_{jt}^{x,k} = \sum_r \left(x_{rt}^k - x_{jt}^k \right)^2$$

- We've already shown that price RC generates curvature so can use this IV as a measure of average curvature:

$$Z_{jt}^{p} = \sum_r \left(\hat{p}_{rt} - \hat{p}_{jt} \right)^2$$

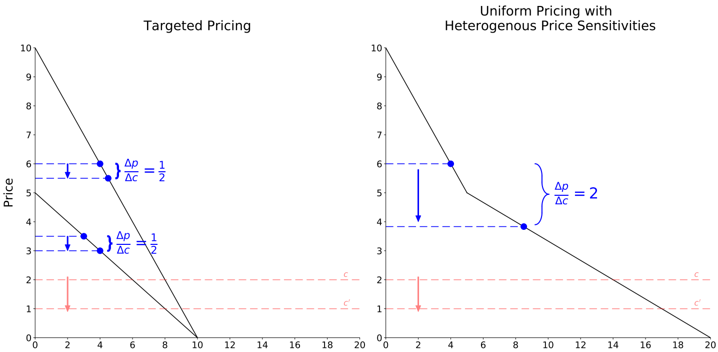
  where $\hat{p}$ comes from hedonic pricing regression using ω_{jt}.

- Identify λ by interacting curvature measure (Z^p) with distribution moments:

$$Z_t^d = Z_t^p \otimes \left\{ \text{inc}_t^{10\%}, \text{inc}_t^{50\%}, \text{inc}_t^{90\%} \right\}.$$

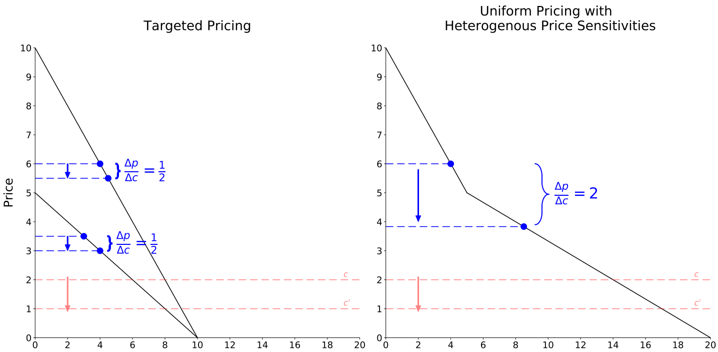
  Idea: Skewness of price interactions determines curvature $\Rightarrow$ interact pass-through measure with moments from the distribution.

Intuition: Heterogeneous Price Sensitivities



- Consider the case of two consumers with linear demand curves of different slope.

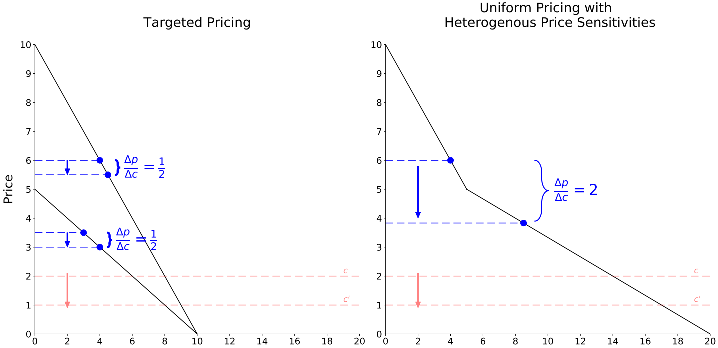
Intuition: Heterogeneous Price Sensitivities



Left Panel

- Suppose monopolist can set prices for each individual.
- Marginal cost is \$2 and decreases by \$1. **How does the firm respond?**
- Firm decreases price by \$0.5 for both consumers.

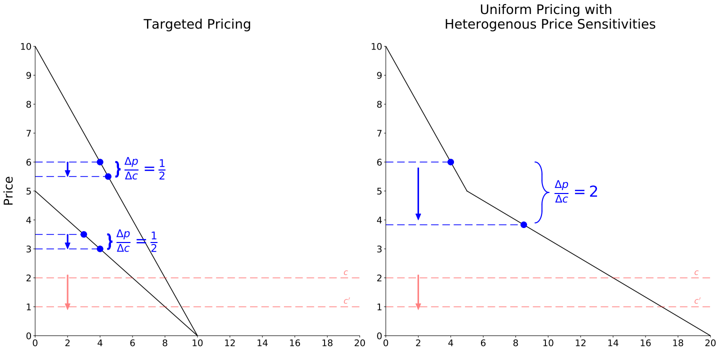
Intuition: Heterogeneous Price Sensitivities



Right Panel

- Constrain the firm to uniform pricing.
- Marginal cost is \$2 and decreases by \$1. **How does the firm respond?**
- Firm decreases price by \$2.0.
- The cost reduction resulted in firm setting a price such that the price-sensitive consumer participates.

Intuition: Heterogeneous Price Sensitivities



Discussion

- Pass-through could be over-shifted w/ uniform pricing + heterogeneous price-sensitivity.
- The effect of a cost shift is different at different price levels!
- Widespread evidence of overshifting and uniform pricing in retail.

Results - Parameters

Scenario	α (varies)		λ (varies)		$\sigma_x = 5$		$\sigma_0 = 5$	
True-Specification	$A.Bias$	$RMSE$	$A.Bias$	$RMSE$	$A.Bias$	$RMSE$	$A.Bias$	$RMSE$
1: log–log	0.003	0.161	0.000	0.000	-0.006	0.072	-0.012	0.231
2: linear–linear	0.001	0.011	-	-	0.015	0.090	-0.082	0.947
3: BC–BC	0.000	0.037	-0.001	0.024	0.006	0.079	-0.001	0.735
4: log–BC	0.331	0.379	0.005	0.006	-0.012	0.070	0.025	0.121
5: linear–BC	-0.031	0.048	-0.060	0.085	0.006	0.091	0.093	1.109
6: BC–log	-15.514	15.612	-	-	0.851	0.947	-2.211	2.218
7: BC-linear	0.248	0.248	-	-	0.015	0.091	-0.272	0.987

1. Scenarios 1–3: MC recovers true parameters when correctly specified.

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1. Scenarios 1–3: MC recovers true parameters when correctly specified.
2. Scenarios 4–5: Estimator recovers true parameters of nested simpler DGPs.

Results - Parameters

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1: log–log	0.003	0.161	0.000	0.000	-0.006	0.072	-0.012	0.231
2: linear–linear	0.001	0.011	-	-	0.015	0.090	-0.082	0.947
3: BC–BC	0.000	0.037	-0.001	0.024	0.006	0.079	-0.001	0.735
4: log–BC	0.331	0.379	0.005	0.006	-0.012	0.070	0.025	0.121
5: linear–BC	-0.031	0.048	-0.060	0.085	0.006	0.091	0.093	1.109
6: BC–log	-15.514	15.612	-	-	0.851	0.947	-2.211	2.218
7: BC-linear	0.248	0.248	-	-	0.015	0.091	-0.272	0.987

1. Scenarios 1–3: MC recovers true parameters when correctly specified.
2. Scenarios 4–5: Estimator recovers true parameters of nested simpler DGPs.
3. Scenarios 6–7: Common (miss-)specifications introduce bias.

Results - Biases Matter

Scenario	*Coeff. Var*		*MAB*		*Corr.*	
True-Specification	DGP	EST.	ε	ρ	(ε, ρ)	$(\hat{\varepsilon}, \hat{\rho})$
1: log–log	-3.81	-3.79	0.00	0.00	0.66	0.66
2: linear–linear	0.00	0.00	0.00	0.00	0.66	0.66
3: BC–BC	-0.57	-0.57	0.00	0.00	-0.47	-0.47
4: log–BC	-3.81	-3.77	0.00	0.00	-0.47	-0.47
5: linear–BC	0.00	-0.11	0.00	-0.01	-0.44	-0.43
6: BC–log	-0.57	-3.77	0.55	-0.69	-0.44	0.63
7: BC-linear	-0.57	0.00	-0.16	0.22	-0.44	-0.43

1. Scenarios 1–3: MC recovers (ε, ρ) when correctly specified.

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1. Scenarios 1–3: MC recovers (ε, ρ) when correctly specified.
2. Scenarios 4–5: Estimator recovers (ε, ρ) of nested simpler DGPs.
3. Scenarios 6–7: Common (miss-)specifications generated biased $(\hat{\varepsilon}, \hat{\rho})$:
   - Biased own- and cross-price elasticities $\Rightarrow$ antitrust implications.
   - Biased curvature $\Rightarrow$ pass-through (*e.g.*, inflation) and trade implications.

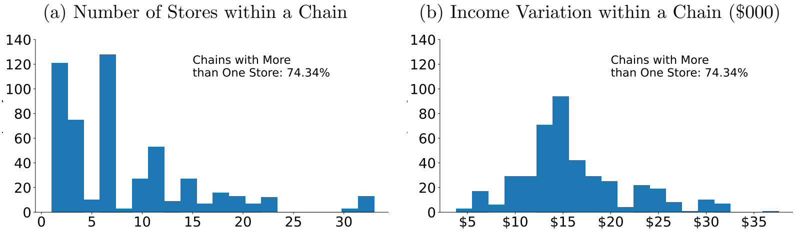
Question and Empirical Strategy

- **Motivation:**
- Increased access to customer data & sophisticated pricing raises concern about distributional implications. – CEA (2015)
- Welfare effects of 3^{rd}-degree price discrimination (3DPD) driven by relative curvature of local demands. – Aguirre, Cowan & Vickers (2010)
- **Research Question:** How does the specification of demand affect our estimate of the consumer welfare implications of 3DPD?
- **Approach:**
- Mixed-Logit demand estimation using store-level RTE cereal data.
- Recover upstream marginal cost of each product based on multi-product firm portfolios under uniform pricing across stores in given market.
- Experiment: Given recovered marginal cost and preferences, allow products' prices to vary by store & recompute equilibrium prices.

IRI: Breakfast Cereal

- Weekly scanner data for ready-to-eat (RTE) cereal from 2007–2011.
- Product defined brand-flavor pair; *e.g.*, Kellogg's Special K Fruit & Yogurt.
- Serving defined as one ounce.
- Potential market identified via milk and paper towels.
  – Backus, Conlon & Sinkinson (2021)
- Focus on products which account for 85% of sales.
- Large markets with geographic spread: Boston (5.2% of revenue), Philadelphia (4.5%), Chicago (4.2%), San Francisco (3.0%), Seattle (2.5%), Houston (2.5%), and St Louis (2.4%). Ind'l level: Eau Clair, Pittsfield.
- Append demographic information matched by Public-Use Microdata (PUMA) region from the American Community Survey (ACS).
  – Most variation across geography, not time.

Chains, Markets Served, & Uniform Pricing



- Many multi-location chains in data.
- Locations differ in income.
- As in Della Vigna & Gentzkow (2017) and Hitsch, Hortacsu, & Lin (2021), uniform pricing prevalent: for median product, chain fixed effects explain 72% of variation in price; market fixed effects 31%.

Specification

- Quasi-linear indirect utility:

$$u_{ijt} = x_j \beta_i^\star + \alpha_i^\star p_{jt} + \xi_{jt} + \epsilon_{ijt}$$

- Characteristic and price random coefficients are defined as

$$\begin{pmatrix} \alpha_i^* \\ \beta_i^* \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} + \Pi D_{il} + \Sigma \nu_{il} \, , \quad \nu_{il} \sim N(0, I_{n+1}) \, ,$$

- Flexible price-interactions:

$$\alpha_i^\star = \alpha + \pi^{\text{kids}} \times \mathbb{1}^{\text{kids}} + \pi^y \times y_i^{(\lambda)}$$

where

$$y_i^{(\lambda)} \equiv \begin{cases} \dfrac{y_i^\lambda - 1}{\lambda} \, , & \lambda \neq 0 \\ \ln\left(y_i\right) \, , & \lambda = 0 \end{cases}$$

Estimation

- Demand-side.
- Estimator:

$$\hat{\theta} = \underset{\theta}{\text{argmin}} \left\{ g(\theta)' W g(\theta) \right\}, \text{ where } g(\theta) = \begin{bmatrix} g^{\mathsf{M}}(\theta) \\ g^{\mathsf{D}}(\theta) \end{bmatrix}$$

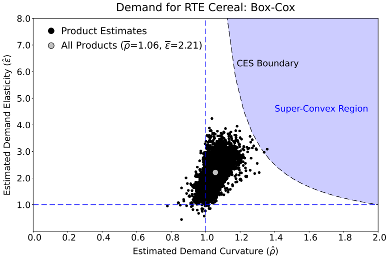
- *BLP* moment conditions:

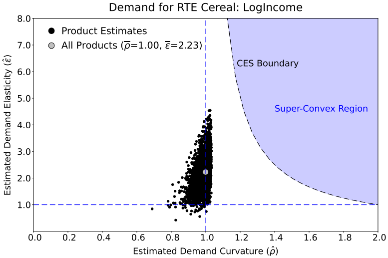
$$g^{\mathsf{M}}(\theta) \equiv E\left[Z' \xi(\theta) \right]$$

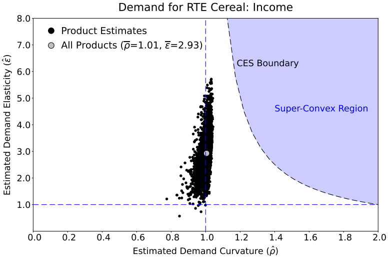
where Z are MC instruments, including Z^d to identify λ.

- Micro moment conditions ($g^{\mathsf{M}}(\theta)$)

1. $E[price|y_i \in Q_k]/E[price|y_i \in Q_1], k = 2, 3, 4$
2. $E[y_i|\text{buy}]$
3. $\text{cov}(\text{kids}, price)$
4. $E[\text{kids}|\text{buy}]$


Demand for RTE Cereal: Box-Cox
Product Estimates
All Products ($\bar{\rho}$=1.06, $\bar{\varepsilon}$=2.21)
CES Boundary
Super-Convex Region
Estimated Demand Elasticity ($\hat{\varepsilon}$)
Estimated Demand Curvature ($\hat{\rho}$)


Demand for RTE Cereal: LogIncome
Product Estimates
All Products ($\bar{\rho}$=1.00, $\bar{\varepsilon}$=2.23)
CES Boundary
Super-Convex Region
Estimated Demand Elasticity ($\hat{\varepsilon}$)
Estimated Demand Curvature ($\hat{\rho}$)


Demand for RTE Cereal: Income
Product Estimates
All Products ($\bar{\rho}$=1.01, $\bar{\varepsilon}$=2.93)
CES Boundary
Super-Convex Region
Estimated Demand Elasticity ($\hat{\varepsilon}$)
Estimated Demand Curvature ($\hat{\rho}$)

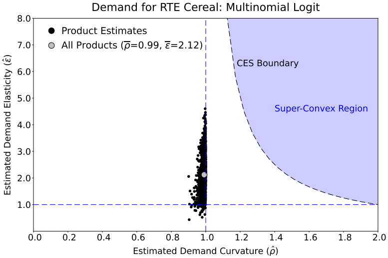

Demand for RTE Cereal: Multinomial Logit
Product Estimates
All Products ($\bar{\rho}$=0.99, $\bar{\varepsilon}$=2.12)
CES Boundary
Super-Convex Region
Estimated Demand Elasticity ($\hat{\varepsilon}$)
Estimated Demand Curvature ($\hat{\rho}$)

Table: IRI: Flexible Demand

	Flexible	Income	Log-Income	MNL
Elasticities				
- Mean	2.14	2.93	1.90	2.16
- Median	2.14	2.91	1.88	2.14
- Stand. Dev.	0.45	0.67	0.44	0.53
- 90%	2.71	3.79	2.47	2.85
- 10%	1.55	2.06	1.34	1.49
Curvature				
- Mean	1.09	1.01	1.03	1.00
- Median	1.08	1.01	1.03	1.00
- Stand. Dev.	0.05	0.02	0.03	0.01
- 90%	1.15	1.02	1.06	1.00
- 10%	1.04	0.98	1.01	0.99

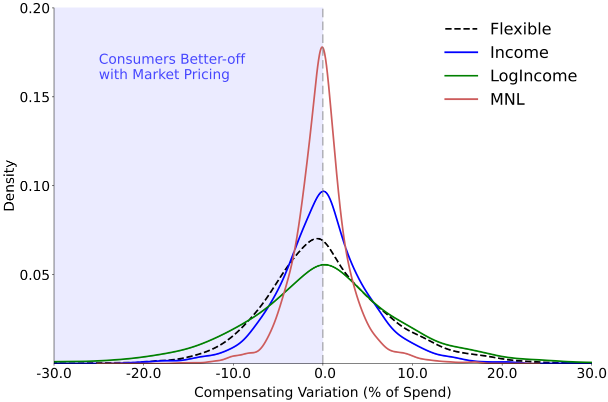
Table: IRI: Matching Consumption Patterns

Moment	Data	Flexible ($\hat{\lambda}=2.31$)	Income ($\lambda=1.00$)	Log-Income ($\lambda=0.00$)	MNL
$\mathbb{E}$[Price|IncomeQ_2]/$\mathbb{E}$[Price|IncomeQ_1]	1.0011	1.0022	1.0128	1.0187	1.0000
$\mathbb{E}$[Price|IncomeQ_3]/$\mathbb{E}$[Price|IncomeQ_1]	1.0087	1.0091	1.0252	1.0250	1.0000
$\mathbb{E}$[Price|IncomeQ_4]/$\mathbb{E}$[Price|IncomeQ_1]	1.0492	1.0498	1.0478	1.0309	1.0000
Corr(Price,Kids)	-0.0149	-0.0149	-0.0132	-0.0164	0.0000
E[Income|Buy]	0.9852	0.9851	0.9852	0.9851	1.0000
E[Kids|Buy]	1.2470	1.2469	1.2435	1.1492	1.0000

Alternative Specifications – Implications

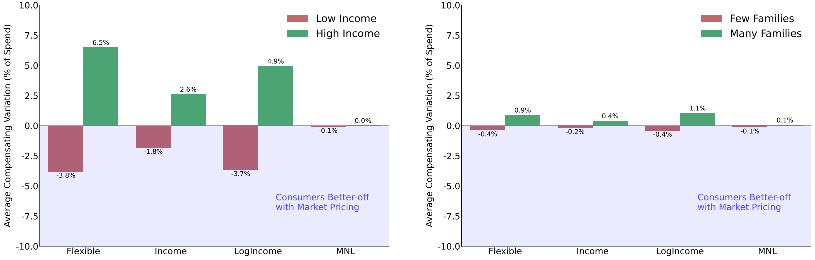
- Consider impact of demand specification on estimated consumer welfare effects of uniform pricing:
  - Assuming firms set price a la multi-product Bertrand-Nash, recover single product MC from observed uniform pricing.
  - Holding fixed estimated MCs and ownership patterns, predict optimal store-level prices and optimal uniform prices for each product.
- Assess welfare implications of uniform pricing, relative to store-level pricing, via compensating variation.
  - $CV > 0 \rightarrow$ consumer benefits from uniform pricing.

Alternative Specifications – Consumer Welfare



- The spread of the distribution on compensating variation follows from each models ability to match the distribution of price sensitivity and the distribution of demand curvature.
- All four specifications predict that on average, consumers are near indifferent between targeted and uniform pricing, but models make different distributional predictions (spread).

Alternative Specifications – Winners & Losers of Uniform Pricing



(A) Income (B) Kids

- We decompose the distributions of winners and losers of uniform pricing by demographic group.
- Allowing for flexibility in the estimation of demand curvature leads to very different sign and magnitude of welfare effects.

Contribution

- We explore the determinants of demand curvature estimates in aggregate discrete choice models.
- We show that a unit-demand BLP-style model can accommodate a wide range of demand curvatures beyond MNL and up to CES.
- We provide a flexible and parsimonious approach to specifying price responsiveness.
  - Implementation uses the famous Box-Cox transform and amounts to adding a single parameter.
  - Identification is straight-forward and intuitive.
- Monte Carlo and empirical results indicate that this flexibility is economically important for policy.

Thank you!

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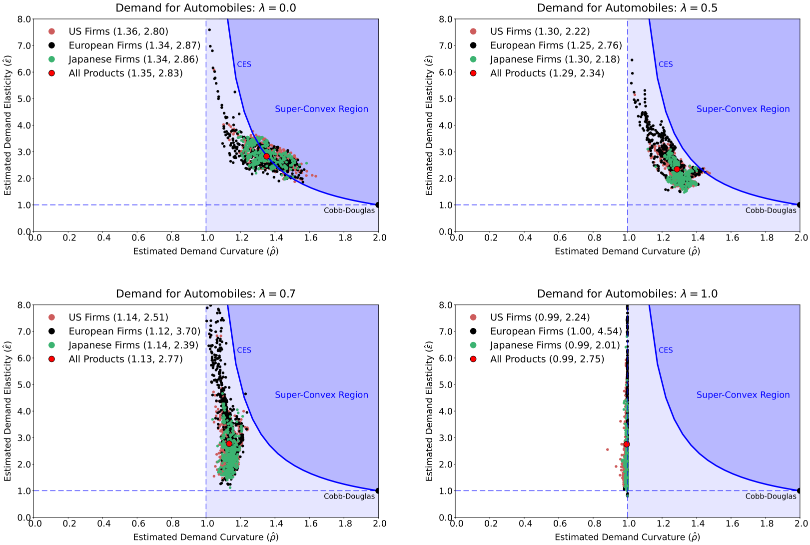
Appendix

BLP99: Effects by Geographic Origin

Return



Nevo's Estimates

Return

Table: Breakfast Cereal: Price Related Estimates

	Means	Std. Dev.	Demographic Interactions ($\pi's$)			Manifold	
Specification	($\alpha's$)	($\sigma's$)	log(Income)	log(Income)^2	Child	ε	ρ
[A]	-62.7299 (14.8032)	3.3125 (1.3402)	588.3252 (270.4410)	-30.1920 (14.1012)	11.0546 (4.1226)	3.62	1.06
[B]	-30.9982 (0.9674)	2.0216 (0.9367)	— —	— —	— —	3.74	0.96
[C]	-53.1367 (12.1023)	— —	444.7281 (209.6548)	-22.3987 (10.7282)	16.3664 (4.7824)	3.60	1.08
[D]	-30.8902 (0.9944)	— —	— —	— —	— —	3.74	0.96

- Notice that price random coefficients are significant but very small relative to the demand slope estimates.
- Demographic interactions are substantial.
- Average elasticity estimates appear robust across different specifications.
- Average curvatures are dangerously close to 1, with very different pass-through rates implications. Do we need a structural model at all?
- Averages say nothing about the distribution of $(\hat{\varepsilon}, \hat{\rho})$.